The (Metaphysical) Foundations of Arithmetic?

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1. Introduction

Not so long ago, Gideon Rosen wrote:

> We say that one class of facts *depends upon* or is *grounded in* another. We say that a thing possesses one property *in virtue of* possessing another, or that one proposition *makes* another true. These idioms are common … but they are not part of anyone's official vocabulary. The general tendency is to admit them for heuristic purposes, where the aim is to point the readers' nose in the direction of some philosophical thesis, but then to suppress them in favor of other, allegedly more hygienic formulations when the time comes to say *exactly* what we mean. The thought is apparently widespread that while these ubiquitous idioms are sometimes convenient they are ultimately too unclear or too confused, or perhaps simply too exotic to feature in our first-class conceptual vocabulary.[i]

It is strange that highly influential papers so rapidly become dated. Very many metaphysicians (though not all)[ii] now include 'grounds', or some related term, in their 'official vocabulary', just as Rosen and others[iii] recommended.

Much of the discussion of grounding so far has focused on very general questions, including:

- What are the relata of the grounding relation,[iv] if it is a relation at all?[v]
- What are the formal properties of the grounding relation (e.g. is it acyclic)?[vi]
- What grounds facts about what grounds what?[vii]

I do not object to these very high-level discussions, but I believe that we metaphysicians should also work in detail on special cases – and currently there are very few such detailed case studies in the literature. When examples of grounding are given, they tend to be either very simple toy cases, or very grand unspecific claims such as 'The normative is grounded by the natural'.

This paper is a thorough discussion of a proposal due independently to Robert Schwartzkopff and Gideon Rosen about what grounds facts involving cardinal numbers. Roughly, the principle is as follows:

> For any properties F and G, if the number of things that have the property F is identical to the number of things that have the property G, then this fact is grounded by the fact that the things that have the property F and the things that have the property G can be paired one-to-one.[viii]

For obvious reasons, I call this the 'Schwartzkopff-Rosen Principle'. The principle is a perfect case study: it is precise enough that it can be investigated in detail, but it is no mere toy case. On the contrary, the Schwartzkopff-Rosen Principle is of considerable philosophical significance, as I shall now briefly explain.

In his discussions of the metaphysics of number, Aristotle rejected the Platonistic claim that numbers are 'prior in substance' to perceptible things, and 'separate' from them; for Aristotle, numbers are nothing 'apart from' perceptible things.[ix] To put this in today's terms, Aristotle's claim was that numbers are grounded by ordinary empirical things – or, alternatively, that arithmetical facts are grounded by empirical facts. This is an appealing idea, but the details of Aristotle's position are less attractive to the typical modern reader. Aristotle's proposal, I believe, was that when a natural number *n* exists, *n* is grounded by each plurality of *n* concrete things: for example, the players in a rugby union team together ground the number fifteen. On this view, it is doubtful that very large natural numbers exist. On the Aristotelian view, it is not clear that the number $10^{10^{10^{10}}}$ exists because it is doubtful that there exists a plurality of $10^{10^{10^{10}}}$ concrete things.

Jonathan Barnes (1985) suggests that Aristotle could have defended the claim that arbitrarily large finite cardinals exist by appeal to the infinite divisibility of matter:

> Physical objects are, in Aristotle's view, infinitely divisible. That fact ensures that, even within the actual finite universe, we shall always be able to find a group of *k* objects, for any *k* … If the universe consisted simply of a single sphere, it would also contain two objects (two hemispheres), three objects (three third-spheres) and so on. We shall never run short of numbers of things …

Most modern readers will be very unhappy with a view on which the existence of $10^{10^{10^{10}}}$ is doubtful, and only slightly less unhappy with a view on which the existence of this number is secured only by appeal to a (presumably contingent) hypothesis about the structure of matter.

As we shall see, by endorsing the Schwartzkopff-Rosen Principle one can preserve the Aristotelian idea that arithmetical facts are grounded by empirical facts, without being stuck with Aristotle's unpalatable conclusions about large numbers.

I proceed as follows. I begin in section two by presenting a 'framework' for the subsequent discussion – that is, I set out my preferred notation and my initial assumptions. In section three I begin my discussion of the Schwartzkopff-Rosen Principle. In sections four, five, six and seven I refine the principle. In section eight I show that the principle implies that the relation of 'partial ground' is not acyclic. Section nine is my conclusion. In an appendix, I discuss ground and second-order quantification.

2. Notation and basic assumptions

Several different frameworks for thinking about ground have been developed. Perhaps the most well-known are those given in Fine 2012, Rosen 2010 and Schaffer forthcoming. I will use Rosen's notation and (with some exceptions) I'll accept his basic assumptions. I do this *not* because I think that Rosen's framework is superior to Fine's or Schaffer's.[x] I take no stand on this question. Rather, I think that what follows could be reconstructed within any of the three frameworks, and I happen to find Rosen's notation easier to handle.

For Rosen, ground is a relation among *facts*. I represent these facts by enclosing sentences in square brackets. For example, '[*Owl*(Barney)]' refers to the fact that Barney is an owl. I will sometimes want to refer to a plurality of facts that share a common form. To do this, I put an open formula between square brackets. For example, here are the facts of the form [*Owl*(x)]:

[*Owl*(Barney)], [*Owl*(Hooter)], [*Owl*(Ruffles)], …

I assume throughout that facts are composite. For example, [*Owl*(Barney)] has as constituents Barney himself, and the property of being an owl (Rosen 2010:115).[xi] I represent the relation of total ground using a left-pointing arrow '←' like so:

$$F \leftarrow \Gamma$$

There are two arguments here. On the left, a single fact (this is the groundED fact); on the right, a set of facts (these are the facts that do the groundING). When stating claims about grounding, I will often omit braces '{' and '}' on the basis that no confusion is likely to result.[xii, xiii] The relation of partial ground is represented by the arrow '⇤' and defined as follows:

For any facts F_0 and F_1:
F_0 ⇤ F_1 iff there is some set Γ such that $F_0 \leftarrow \Gamma$ and $F_1 \in \Gamma$.

I will also sometimes describe 'chains' of ground, thus:

(a)	[α]	←	[β]	←	[γ]	←	[δ]	←	…
(b)	[α]	⇤	[β]	⇤	[γ]	⇤	[δ]	⇤	…

Such claims are to be interpreted in the obvious way:

(a)	[α] ← {[β]};	[β] ← {[γ]};	[γ] ← {[δ]};	…
(b)	[α] ⇤ [β];	[β] ⇤ [γ];	[γ] ⇤ [δ];	…

We may assume that the relation of total ground is transitive, in the following sense:[xiv]

If $[p]\leftarrow\{[q]\}\cup\Gamma$ and $[q]\leftarrow\Delta$, then $[p]\leftarrow\Gamma\cup\Delta$.

It follows that the relation of partial ground is transitive in a more familiar sense:

For all F_0, F_1 and F_2, If F_0⇤F_1 and F_1⇤F_2 then F_0⇤F_2.

Fine and Rosen have between them suggested many very plausible general principles about what grounds logically complex facts. They claim that a conjunctive fact is totally grounded by its conjuncts taken together. If Barney is wise and nocturnal, then this fact is totally grounded by [*Wise*(Barney)] and [*Nocturnal*(Barney)]. They also claim that a disjunctive fact is totally grounded by each of its true disjuncts: if Barney is a barn owl, then this fact totally grounds the fact that Barney is a either a barn owl or a tawny owl. Conditionals, as always, are harder. However we can avoid these problems by working exclusively with the *material* conditional: $\ulcorner (\alpha \rightarrow \beta) \urcorner$ abbreviates $\ulcorner (\neg\alpha \vee \beta) \urcorner$. I assume too that $\ulcorner (\alpha \leftrightarrow \beta) \urcorner$ abbreviates $\ulcorner (\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha) \urcorner$. I suppose that in all cases $[\neg\neg\alpha]$ is totally grounded by $[\alpha]$; some readers will wish to identify these two facts – nothing hangs on this issue.

Putting all this together, we obtain the following collection of rules:

(¬ rule)	If α, then [¬¬α]←[α].
(∧ rule)	If (α∧β), then [α∧β]←[α], [β].
	If ¬α, then [¬(α∧β)]←[¬α] and [¬(β∧α)]←[¬α].
(∨ rule)	If α, then [α∨β]←[α] and [β∨α]←[α].
	If ¬(α∨β), then [¬(α∨β)]←[¬α], [¬β].
(→ rule)	If β then [α→β]←[β].
	If ¬α then [α→β]←[¬α].
	If α and ¬β, then [¬(α→β)]←[α], [¬β].
(↔ rule)	If α and β, then [α↔β]←[α], [β].
	If ¬α and ¬β, then [α↔β]←[¬α], [¬β].
	If α and ¬β, then [¬(α↔β)]←[α], [¬β].
	If ¬α and β, then [¬(α↔β)]←[¬α], [β].

Facts involving quantification are more difficult. It is natural to suppose that [*Owl*(Barney)] grounds [∃x *Owl*(x)]. It is also natural to suppose that a universal fact is grounded by all of its instances together: [∀x Fx] ← [Fa], [Fb], [Fc], … . Putting this together, we obtain:

(Naïve 1st ∀ rule)
If ∀x φ(x), then [∀x φ(x)] is totally grounded by the set of facts of the form [φ(x)].
If ¬∀x φ(x), then for any fact F of the form [¬φ(x)], [¬∀x φ(x)] ← F.

(Naïve 1st ∃ rule)
If ∃x φ(x), then for any fact F of the form [φ(x)], [∃x φ(x)] ← F.
If ¬∃x φ(x), then [¬∃x φ(x)] is totally grounded by the set of facts of the form [¬φ(x)].

Fine and Rosen reject these proposals, because they conflict with 'grounding necessitarianism', as follows:

> Suppose that some fact F is totally grounded by some set of facts Γ. Then there is no metaphysically possible world at which each element of Γ obtains and F doesn't obtain.

This principle is very often assumed without argument in the literature: Bliss and Trogdon (2014) describe it as the 'default view'. I will discuss later whether we have good reason to accept the principle.

To see the conflict between grounding necessitarianism and the naïve 1st $\forall$ rule, let F be the property *being a twentieth century French monarch* and suppose that a, b, c, d, … are all the objects that there are. Now the naïve 1st $\forall$ rule implies:

$$[\forall x\, \neg Fx] \leftarrow [\neg Fa], [\neg Fb], [\neg Fc], [\neg Fd], \ldots$$

However, there presumably exists a possible world at which [¬Fa], [¬Fb], [¬Fc], [¬Fd], … obtain but in which some *other* object, distinct from a, b, c, d, … is a twentieth century French monarch. This violates grounding necessitarianism.

Fine and Rosen seek to avoid the problem by saying that [∀x ¬Fx] is also partially grounded by 'the totality fact' – the fact that a, b, c, d, … are all the objects:

$$[\forall x\, \neg Fx] \leftarrow \text{The totality fact}, [\neg Fa], [\neg Fb], [\neg Fc], [\neg Fd], \ldots$$

More generally:[xv]

(Sophisticated 1st $\forall$ rule)
If $\forall x\, \varphi(x)$, then $[\forall x\, \varphi(x)]$ is totally grounded by the set that contains the totality fact together with all facts of the form $[\varphi(x)]$.
If $\neg\forall x\, \varphi(x)$, then for any fact F of the form $[\neg\varphi(x)]$, $[\neg\forall x\, \varphi(x)] \leftarrow F$.

(Sophisticated 1st $\exists$ rule)
If $\exists x\, \varphi(x)$, then for any fact F of the form $[\varphi(x)]$, $[\exists x\, \varphi(x)] \leftarrow F$.
If $\neg\exists x\, \varphi(x)$, then $[\neg\exists x\, \varphi(x)]$ is totally grounded by the set that contains the totality fact together with all facts of the form $[\neg\varphi(x)]$.

This preserves grounding necessitarianism, but at a cost. The inclusion of the 'totality fact' in the grounds for universal and negative existential facts seems *ad hoc.* And it gets worse. Consider what the 'totality fact' is. Rosen suggests that the totality fact is:

$$[\forall x(x=a \vee x=b \vee x=c \vee \ldots)] \qquad \text{(where a, b, c … are all the things that there are)}$$

This natural proposal is problematic. Rosen assumes that the relation of partial ground is acyclic,[xvi] which forces Rosen to say that the totality fact itself is an exception to the normal rule governing universal facts. He concludes that the totality fact is ungrounded.[xvii] Fine (2012) suggests that we avoid the problem by rejecting the identification of the totality fact with [$\forall$x(x=a $\vee$ x=b $\vee$ x=c $\vee$ …)]. His alternative position is that the totality fact is *sui generis*, which seems dangerously obscure. So both the naïve and the sophisticated rules for facts involving first-order quantification are problematic. I will suggest a solution to this problem in section six; until then, I leave the issue open.

Still more difficult are facts involving second-order quantification. I assume that the second-order variables range over properties and relations, which I represent using this notation:

$\langle\lambda x : \textit{Feathery}(x) \wedge \textit{Wise}(x)\rangle$	the property *being feathery and wise*.
$\langle\lambda x : \exists y(\textit{Owl}(y) \wedge \textit{Smaller}(x,y))\rangle$	the property *being smaller than some owl*.
$\langle\lambda xy : \textit{Sibling}(x, y) \wedge \textit{Male}(x)\rangle$	the relation that x bears to y iff x is a male sibling of y.

I will assume that it is grammatical to use these λ-expressions as predicates, like so:

$\langle\lambda x : \textit{Feathery}(x) \wedge \textit{Wise}(x)\rangle$Barney

Now it seems obvious that [$\langle\lambda x: \textit{Feathery}(x) \wedge \textit{Wise}(x)\rangle$Barney] is totally grounded by [*Feathery*(Barney) $\wedge$ *Wise*(Barney)].[xviii] That is, Barney has the conjunctive property *being feathery and wise* because he is feathery and he is wise. More generally:

(λ rule)
If $\langle\lambda x : \varphi(x)\rangle t$, then $[\langle\lambda x: \varphi(x)\rangle t]\leftarrow[\varphi(t)]$.
If $\neg\langle\lambda x : \varphi(x)\rangle t$, then $[\neg\langle\lambda x : \varphi(x)\rangle t]\leftarrow[\neg\varphi(t)]$.[xix]

This principle is easily generalized to cover relations.

Now let's turn to grounding and second-order quantification. As with the first-order quantifiers, there's a choice to be made between the 'naïve' and 'sophisticated' approaches:

(Naïve 2^{nd} $\forall$ rule)

If $\forall X\ \varphi(X)$, then $[\forall X\ \varphi(X)]$ is totally grounded by the set of facts of the form $[\varphi(X)]$.

If $\neg\forall X\ \varphi(X)$, then for any fact F of the form $[\neg\varphi(X)]$, $[\neg\forall X\ \varphi(X)] \leftarrow F$.

(Naïve 2^{nd} $\exists$ rule)

If $\exists X\ \varphi(X)$, then for any fact F of the form $[\varphi(X)]$, $[\exists X\ \varphi(X)] \leftarrow F$.

If $\neg\exists X\ \varphi(X)$, then $[\neg\exists X\ \varphi(X)]$ is totally grounded by the set of facts of the form $[\neg\varphi(X)]$.

(Sophisticated 2^{nd} $\forall$ rule)

If $\forall X\ \varphi(X)$, then $[\forall X\ \varphi(X)]$ is totally grounded by the set that contains the totality fact together with all facts of the form $[\varphi(X)]$.

If $\neg\forall X\ \varphi(X)$, then for any fact F of the form $[\neg\varphi(X)]$, $[\neg\forall X\ \varphi(X)] \leftarrow F$.

(Sophisticated 2^{nd} $\exists$ rule)

If $\exists X\ \varphi(X)$, then for any fact F of the form $[\varphi(X)]$, $[\exists X\ \varphi(x)] \leftarrow F$.

If $\neg\exists X\ \varphi(X)$, then $[\neg\exists X\ \varphi(X)]$ is totally grounded by the set that contains the totality fact together with all facts of the form $[\neg\varphi(X)]$.

I discuss some slightly technical issues to do with second-order quantification in an appendix – but these issues need not trouble us as we discuss 'numerical facts' – that is, facts that have cardinal numbers as constituents.

3. The Schwartzkopff-Rosen Principle

Having introduced my 'framework', I am now ready to describe the Schwartzkopff-Rosen Principle. First, some notation. The symbol '#' stands for a function which maps each property to a cardinal number – specifically, the number of things which have the property. For example, if '*WelshCity*' is a predicate true of all and only the cities in Wales, '#*WelshCity*' refers to the number of cities in Wales. Similarly, if '*FlamingoSpecies*' is a predicate true of all and only the species of flamingo, '#*FlamingoSpecies*' refers to the number of species of flamingo.

According to a group of philosophers known as the 'neo-fregeans',[xx] the operator '#' can be defined using 'Hume's Principle', which is this:

$$\forall X \forall Y(\#X=\#Y \leftrightarrow \exists W\ X \sim_W Y)$$

Loosely speaking, '$X \sim_W Y$' means that the relation W pairs the Xs and the Ys 'one-to-one', so that each X is paired with exactly one Y and each Y is paired with exactly one X. More formally, '$X \sim_W Y$' abbreviates the conjunction of the following four conditions:

$\text{Function}_{X,Y}(W)$:	$\forall x(Xx \rightarrow \forall y_1(Yy_1 \rightarrow \forall y_2(Yy_2 \rightarrow (Wxy_1 \wedge Wxy_2 \rightarrow y_1=y_2))))$
$\text{Injective}_{X,Y}(W)$:	$\forall y(Yy \rightarrow \forall x_1(Xx_1 \rightarrow \forall x_2(Xx_2 \rightarrow (Wx_1y \wedge Wx_2y \rightarrow x_1=x_2))))$
$\text{Total}_{X,Y}(W)$:	$\forall x(Xx \rightarrow \exists y(Yy \wedge Wxy))$
$\text{Onto}_{X,Y}(W)$:	$\forall y(Yy \rightarrow \exists x(Xx \wedge Wxy))$

I will often omit the subscripts, since it will usually be obvious from the context which properties are relevant. Let's look at an example. Here is an instance of Hume's Principle:

$$\#\textit{FlamingoSpecies} = \#\textit{WelshCity} \leftrightarrow \exists W\ \textit{FlamingoSpecies} \sim_W \textit{WelshCity}$$

In English: the number of species of Flamingo is identical to the number of cities in Wales just in case there is a relation which pairs the cities in Wales with the species of Flamingo one-to-one. Now it turns out that there *is* a relation which pairs the species of flamingo with the Welsh cities:

Greater Flamingo	←————————————→	Bangor
Lesser Flamingo	←————————————→	Cardiff
Chilean Flamingo	←————————————→	Newport
Andean Flamingo	←————————————→	St Davids
James's Flamingo	←————————————→	Swansea
American Flamingo	←————————————→	St Asaph

So Hume's Principle tells us that the number of species of flamingo is indeed identical with the number of cities in Wales.

We need not take a stand here on the question of whether Hume's Principle is a good *definition* of '#': we're interested in metaphysical grounding, not metasemantics or epistemology. The Schwartzkopff-Rosen Principle is a natural and elegant modification of Hume's Principle:

For all X and Y, if #X=#Y, then [#X=#Y] ← [∃W X $\sim_W$ Y].
For all X and Y, if #X≠#Y, then [#X≠#Y] ← [¬∃W X $\sim_W$ Y].[xxi, xxii]

For example:

[#*FlamingoSpecies* = #*WelshCity*] ← [∃W *FlamingoSpecies* $\sim_W$ *WelshCity*]

This will appeal to the Aristotelian. *Prima facie,* what we have here is a numerical fact (viz. [#*FlamingoSpecies* = #*WelshCity*]) totally grounded by a non-numerical fact (viz. [∃W *FlamingoSpecies* $\sim_W$ *WelshCity*]).[xxiii]

Despite its simplicity, the Schwartzkopff-Rosen Principle promises to give us an account of what grounds a wide range of arithmetical facts, as I shall now explain.[xxiv] One of the achievements of the philosophers of the neo-fregean school is to have shown that many of the standard terms in the theory of cardinals can be defined explicitly using the '#' operator. For example:

Cardinal(x)	abbreviates	∃X(x = #X)
0	abbreviates	#⟨λx : x ≠ x⟩
1	abbreviates	#⟨λy : y = 0⟩
2	abbreviates	#⟨λz : z = 0 ∨ z = 1⟩
Sum(x, y, z)	abbreviates	∃X ∃Y (¬∃w(Xw ∧ Yw) ∧ x=#X ∧ y=#Y ∧ z=#⟨λw : Xw ∨ Yw⟩)

Now consider the fact that 2 is the sum of 1 and 1:

[*Sum*(1, 1, 2)]

Given the definitions above, this is just:

[∃X ∃Y (¬∃w(Xw ∧ Yw) ∧ #⟨λy : y = #⟨λx : x ≠ x⟩⟩ = #X ∧ #⟨λy : y = #⟨λx : x ≠ x⟩⟩ = #Y ∧ #⟨λz : z = #⟨λx : x ≠ x⟩ ∨ z = #⟨λy : y = #⟨λx : x ≠ x⟩⟩⟩=#⟨λw : Xw ∨ Yw⟩)]

and it would seem that we will be able to give an account of what grounds this fact using the Schwartzkopff-Rosen Principle, together with the various other principles of ground discussed above. At first glance, the Schwartzkopff-Rosen Principle is no more than an account of what grounds facts about identity and non-identity among cardinals. But at second glance, the principle promises to yield an account of what grounds a very wide variety of arithmetical facts.

4. The individuation of arithmetical facts

In this section, I address the hairy question of how arithmetical facts are individuated. For example, are facts (a) and (b) identical or distinct?

(a) [#*FlamingoSpecies* = #*WelshCity*]

(b) [#*FlamingoSpecies* = #*FlamingoSpecies*]

It turns out that this is a tough question for the proponent of the Schwartzkopff-Rosen Principle. There are two natural positions on the topic, and both of them turn out to be problematic.

The two natural positions are these. According to the 'fine view', the fact [… #X …] has the property X as a constituent. On this view, the constituents of (a) are the identity relation, the property *being a Welsh city,* the property *being a species of Flamingo* and the numbering function; (b) on the other hand does not have the property *being a Welsh city* as a constituent. Therefore, (a) and (b) have different constituents, and so they're distinct. According to the 'coarse view', the fact [… #X …] has as a constituent a certain object – viz. the number of Xs, whatever that happens to be – but not necessarily the property X. On this view, (a) and (b) are the same fact, a fact of the form [x=x].

Both positions are problematic. According to the coarse view, the Schwartzkopff-Rosen Principle only gives us an account of what grounds the facts of the form [x=x] (where x is a number) and [¬x=y] (where x and y are distinct numbers). Now facts of these kinds are necessary, so on the coarse view the Schwartzkopff-Rosen Principle fails to give us an account of what grounds contingent numerical facts. Thus, the coarse view is problematic.

The fine view is similarly problematic. According to the fine view, the Schwartzkopff-Rosen Principle gives us only an account of what grounds facts of the form [#X=#Y] or [¬#X=#Y], it does not give us an account of what grounds facts of the form [x=x] (where x is a number) or [¬x=y] (where x and y are distinct numbers).

My proposal is this. The proponent of the Schwartzkopff-Rosen Principle must reject the coarse view because it leaves her without an account of what grounds certain contingent arithmetical facts. So she must accept the fine view. She should then *extend* the Schwartzkopff-

Rosen Principle so that it covers facts of the form [x=x] (where x is a number) and [¬x=y] (where x and y are distinct numbers).

There are several ways of doing this. The simplest and most natural is this:[xxv]

For all X, x, Y and y, if #X=x, #Y=y and x=y then each one of the facts [#X=#Y], [#X=x], [y=#Y] and [x=y] is totally grounded by [∃W X $\sim_W$ Y].

For all X, x, Y and y, if #X=x, #Y=y and x≠y then each one of the facts [#X≠#Y], [#X≠y], [x≠#Y] and [x≠y] is totally grounded by [¬∃W X $\sim_W$ Y].

From now on, when I use the title 'the Schwartzkopff-Rosen Principle' it is this extended version of the principle that I have in mind.

My new version of the Schwartzkopff-Rosen Principle has a consequence which some will find surprising. Consider the number six. The fact that *that very number* is identical with itself, according to this proposal, is grounded by the fact that the Welsh cities and the species of flamingo can be put into one-to-one correspondence. Now it may seem strange to say that a purely mathematical fact is grounded by facts about flamingoes and cities. However, if we accept the broadly Aristotelian idea that mathematical objects are nothing 'apart from' perceptible objects, such consequences are only to be expected.

5. The regress

Now we've dealt with the question of how arithmetical facts are individuated, we can start to apply the Schwartzkopff-Rosen Principle. Let's start with a simple case, [0=0]. What grounds this fact? In order to apply the Schwartzkopff-Rosen Principle, we need to choose a property which is not instantiated. F=⟨λx : x≠x⟩ is an obvious choice. Then we can begin:

[0=0] ← [∃W F $\sim_W$ F]

We now need to identify a relation which witnesses this existential generalisation. The identity relation, R=⟨λuv: u=v⟩ is as good a choice as any. Then we can continue:

[∃W F $\sim_W$ F]
← [F $\sim_R$ F] (2nd ∃ rule)
= [Function(R) ∧ Injective(R) ∧ Total(R) ∧ Onto(R)] (Def^n of '∼')
← [Function(R)], [Injective(R)], [Total(R)], [Onto(R)] (∧ rule, repeated)

These four facts need to be considered separately. Let's start with [Total(R)], which is:

$[\forall x(Fx \rightarrow \exists y\ (Fy \wedge Rxy))]$

This is a universal generalisation, and so it is partially grounded by each of its instances. This includes:

$[F1 \rightarrow \exists y\ (Fy \wedge R1y)]$

We continue:

$[F1 \rightarrow \exists y\ (Fy \wedge R1y)]$		
$\leftarrow$	$[\neg F1]$	($\rightarrow$ rule)
$=$	$[\neg\langle\lambda x\text{: } x\neq x\ \rangle 1]$	(Def^n of 'F')
$\leftarrow$	$[\neg\neg 1=1]$	(λ rule)
$\leftarrow$	$[1=1]$	($\neg$ rule)

We're far from having fully investigated the grounds of [0=0], but we've already made a significant discovery: it turns out that, according to our theory, the arithmetical fact [0=0] is partially grounded by another arithmetical fact, [1=1].

Let's leave our investigation of the grounds of [0=0] incomplete, and take a look at the grounds of [1=1]. Let $G=\langle\lambda x : x=0\rangle$ and let $S=\langle\lambda uv : u=0 \wedge v=0\rangle$ then:

$[1=1]$		
$\leftarrow$	$[\exists W\ G \sim_W G]$	(S-R Principle)
$\leftarrow$	$[G \sim_S G]$	(2nd $\exists$ rule)
$=$	[Function(S) $\wedge$ Injective(S) $\wedge$ Total(S) $\wedge$ Onto(S)]	(Def^n of '$\sim$')
$\leftarrow$	[Function(S)], [Injective(S)], [Total(S)], [Onto(S)]	($\wedge$ rule, repeated)

Again, let's focus on [Total(S)]:

[Total(S)]		
=	$[\forall x(Gx \rightarrow \exists y\,(Gy \wedge Sxy))]$	(Def^n of 'Total')
⇤	$[G2 \rightarrow \exists y\,(Gy \wedge S2y)]$	(1st $\forall$ rule)
⇤	$[\neg G2]$	($\rightarrow$ rule)
=	$[\neg\langle\lambda x : x{=}0\rangle 2]$	(Def^n of 'G')
⇤	$[\neg 2{=}0]$	

We've now seen that the theory implies that [0=0] ⇤ [1=1] ⇤ [¬2=0]. Now let's think about what grounds [¬2=0]. Let $H{=}\langle\lambda x{:}\ x{=}0 \vee x{=}1\rangle$ and recall that R is the identity relation and that $F{=}\langle\lambda x : x{\neq}x\rangle$; then:

$[\neg 2{=}0]$		
⇤	$[\neg\exists W\ H \sim_W F]$	(the S-R Principle)
⇤	$[\neg\ H \sim_R F]$	(2nd $\exists$ rule)
=	$[\neg(\text{Function}(R) \wedge \text{Injective}(R) \wedge \text{Total}(R) \wedge \text{Onto}(R))]$	(Def^n of '$\sim$')
⇤	$[\neg\text{Total}(R)]$	($\wedge$ rule, repeated)
=	$[\neg\forall x(Hx \rightarrow \exists y\,(Fy \wedge Rxy))]$	(Def^n of 'Total')
⇤	$[\neg(H1 \rightarrow \exists y\,(Fy \wedge R1y))]$	(1st $\forall$ rule)
⇤	$[\neg\exists y\,(Fy \wedge R1y)]$	($\rightarrow$ rule)
⇤	$[\neg(F3 \wedge R13)]$	(1st $\exists$ rule)
⇤	$[\neg F3]$	($\wedge$ rule)
=	$[\neg\langle\lambda x : \neg x{=}x\rangle 3]$	(Def^n of 'F')
⇤	$[\neg\neg 3{=}3]$	(λ rule)
=	$[3{=}3]$	($\neg$ rule)

We've now seen that the theory implies that [0=0] ⇤ [1=1] ⇤ [¬2=0] ⇤ [3=3]. I could go on, but by now it should be clear that we've seen the start of an unending regress. Is this problematic? It has been claimed the relation of partial ground must be 'well-founded' – that is, there can be no infinite chain F_0 ⇤ F_1 ⇤ F_2 ⇤ F_3 ⇤ … .[xxvi] It is not, however, entirely clear to me that this claim is well-motivated.

As Rosen (2010, pg. 116) writes:

> We should not assume that [the grounding relation] is well founded. That is a substantive question. It may be natural to suppose that every fact ultimately depends on an array of basic facts, which in turn depend on nothing. But it might turn out, for all we know, that the facts about atoms are grounded in facts about quarks and electrons, which are in turn grounded in facts about 'hyperquarks' and 'hyperelectrons' and so on *ad infinitum*. So we should leave it open that there might be an infinite chain of facts $[p] \leftarrow [q] \leftarrow [r] \leftarrow \ldots$ [xxvii]

We do not need to settle this dispute here, because I think it clear that even if you do not accept in general that the relation of partial ground is well founded, you should agree that the regresses generated by the Schwartzkopff-Rosen Principle are vicious. Consider the fact $[\#F \neq \#G]$, for some F and G of your choosing. According to the Schwartzkopff-Rosen Principle, this fact is grounded by $[\neg\exists W\, F \sim_W G]$. And according to the 2^{nd} $\exists$ rule, this latter fact is grounded by a set containing *all* facts of the form $[\neg\, F \sim_W G]$ (and also, perhaps, a totality fact). Now this set includes facts involving some rather exotic binary relations – and this leads to problems. Consider for example the case $F = \langle \lambda x\colon x=0 \rangle$, $G = \langle \lambda x\colon x=0 \vee x=1 \rangle$ and $R = \langle \lambda uv : (u=0 \wedge v=0) \vee (u=0 \wedge v=1 \wedge \neg\varphi) \rangle$ where φ is some arbitrary truth. Then:

$[\neg\#F=\#G]$

⇤	$[\neg\exists W\, F \sim_W G]$	(the S-R Principle)
⇤	$[\neg\, F \sim_R G]$	(2^{nd} $\exists$ rule)
⇤	$[\neg \mathrm{Onto}(R)]$	(Def^{n} of '$\sim$', and the $\wedge$ rule)
=	$[\neg\forall y(Gy \rightarrow \exists x\, (Fx \wedge Rxy))]$	(Def^{n} of 'Onto')
⇤	$[\neg(G1 \rightarrow \exists x\, (Fx \wedge Rx1))]$	(1^{st} $\forall$ rule)
⇤	$[\neg\exists x\, (Fx \wedge Rx1))]$	($\rightarrow$ rule)
⇤	$[\neg(F0 \wedge R01)]$	(1^{st} $\exists$ rule)
⇤	$[\neg R01]$	($\wedge$ rule)
=	$[\neg\langle \lambda uv\colon (u=0 \wedge v=0) \vee (u=0 \wedge v=1 \wedge \neg\varphi) \rangle 01]$	(Def^{n} of 'R')
⇤	$[\neg((0=0 \wedge 1=0) \vee (0=0 \wedge 1=1 \wedge \neg\varphi))]$	(λ rule)
⇤	$[\neg(0=0 \wedge 1=1 \wedge \neg\varphi))]$	($\vee$ rule)
⇤	$[\neg\neg\varphi]$	($\wedge$ rule)
⇤	$[\varphi]$	($\neg$ rule)

What we've shown is that, given the Schwartzkopff-Rosen Principle, *every fact* partially grounds [$\neg$#F=#G]. This is surely not credible.

I suggest that these problems have two sources. Consider an instance of the Schwartzkopff-Rosen Principle:

$$[\#F=\#G] \leftarrow [\exists W\ F \sim_W G]$$

First, following tradition I have defined '$F \sim_W G$' as a conjunction of universal generalizations. Making standard assumptions about grounding and quantification, it follows that [#F=#G] is grounded by facts about all the objects that there are – without restriction. Second, we're supposing that the second-order quantifier '∃W' ranges over all binary relations without restriction – including 'exotic' relations. To avoid these problems, then, the proponent of the Schwartzkopff-Rosen Principle needs an account of restricted quantification.

6. Skiles on restricted quantification

I'll use the following syntax for restricted universal quantification:

$(\forall x: \alpha(x))\ \beta(x)$ $(\exists x: \alpha(x))\ \beta(x)$

For example:

(1) $(\forall x: \mathit{Owl}(x))\ \mathit{Wise}(x)$ (2) $(\exists x: \mathit{Owl}(x))\ \mathit{Nocturnal}(x)$

(1) means *every owl is wise;* (2) means *some owl is nocturnal*. (1) and (2) are not to be confused with (1*) and (2*):

(1*) $\forall x(\mathrm{Owl}(x) \rightarrow \mathrm{Wise}(x))$ (2*) $\exists x(\mathrm{Owl}(x) \wedge \mathrm{Nocturnal}(x))$

When you utter (1), you say of the owls that they are wise. When you utter (1*), you say, of each thing that exists, that if it is an owl then it is wise. When you utter (2), you say of the owls that among them is a nocturnal thing. When you utter (2*) you say of all the things that exist that among them is a nocturnal owl (Belnap 1970).

We already have an account of what grounds facts that involve unrestricted first-order quantification, as in (1*) and (2*), but we don't yet have an account of what grounds facts that involve restricted first-order quantification, as in (1) and (2).

Alex Skiles (2014) provides such an account. For universal generalizations, Skiles' idea is that [(∀x: α(x)) β(x)] is grounded by [β(a)], [β(b)], [β(c)] … where a, b, c … are all the things in the restricted range of quantification. For example, consider the fact that every twentieth century Dutch monarch was female. Writing 'M' for the property *being a twentieth century Dutch Monarch* and 'F' for the property *being female,* Skiles' rule implies:

[(∀x: M(x)) F(x)] ← [F(Wilhelmina)], [F(Juliana)], [F(Beatrix)]

This is a very natural and plausible proposal. For restricted existential generalizations, the proposal is even simpler: Skiles claims that, for example, [(∃x: *President*(x)) *Bearded*(x)] is totally grounded by [*Bearded*(Lincoln)]. More generally:

(Skiles' rule)
If (∀x: α(x)) β(x), then [(∀x: α(x)) β(x)] is totally grounded by the set of facts of the form [β(x)], where x has the property ⟨λx : α(x)⟩.
If α(t) and ¬β(t), then [¬(∀x: α(x)) β(x)] ← [¬β(t)].

If α(t) and β(t), then [(∃x: α(x)) β(x)] ← [β(t)].
If ¬(∃x: α(x)) β(x) , then [¬(∃x: α(x)) β(x)] is totally grounded by the set of facts of the form [¬β(x)], where x has the property ⟨λx : α(x)⟩.

Skiles only considers first-order restricted quantification, but the extension of his rules to the second-order is straightforward.

Skiles's principles are very natural and plausible. They do, however, have a couple of surprising consequences – consequences which some will find objectionable. Consider this fact:

[(∀x: *ChildofSimoneDeBeauvoir*(x)) *Tall*(x)]

According to Skiles' proposal, this fact is grounded by the set of all facts of the form [*Tall*(x)], where x is a child Simone de Beauvoir. However, as it happens, Simone de Beauvoir had no children, so this set is empty. So Skiles' proposal implies that this fact is grounded by the empty set. This may seem peculiar: many people have the intuition that a fact cannot be grounded by the empty set. However, Kit Fine has rejected this widespread intuition. Fine distinguishes *fundamental facts* (facts which are not grounded) from facts which are zero-grounded – i.e. grounded by the empty set (Fine 2012). Those who find the notion of zero-grounding mysterious may wish to reject Skiles' rule for this reason.[xxviii] However, we shall soon see that this feature of Skiles' account has its attractions.[xxix]

The second surprising consequence of Skiles' account is that it is inconsistent with grounding necessitarianism. To return to my example, the proposition that every twentieth-century Dutch monarch is female is not necessitated by the proposition that Wilhelmina is female, the proposition that Juliana is female, and the proposition that Beatrix is female. Skiles infers that we should reject grounding necessitarianism. To assess Skiles' position, we should take a look at the motivations for grounding necessitarianism.

Very often, necessitarianism is presented without defence. (See for example Rosen 2010 and Fine 2012). Now I agree that the principle has some intuitive appeal: but Skiles' account of restricted quantification has intuitive appeal too, and it's not obvious that we should reject the latter intuition to preserve the former.[xxx]

There's a better reason for endorsing grounding necessitarianism. One of the motivations for introducing the notion of grounding in the first place was that grounding relations are supposed to explain supervenience relations. Jaegwon Kim (1990) put it well:

> Supervenience itself is not an explanatory relation. It is not a "deep" metaphysical relation; rather, it is a "surface" relation that reports a pattern of property covariation, suggesting the presence of an interesting dependency relation that might explain it.

For example, a physicalist in the philosophy of mind may wish to explain the supervenience of the mental on the physical by saying that mental facts are grounded by physical facts. Similarly, a naturalist in metaethics may wish to explain the supervenience of the normative on the natural facts by saying that normative facts are grounded by natural facts. But explanations of this kind, it seems, rely on grounding necessitarianism.

There is, however, a way in which we can both have our cake and eat it: we can accept Skiles' account of restricted quantification without losing our ability to use the notion of grounding to explain supervenience relations. The solution relies on the notion of *enabling*, which I take from Jonathan Dancy (2004, ch. 3).

Suppose that Icard promised to feed your cat, and that in consequence he is morally obliged to feed him:

(a) [Icard promised to feed Oscar.]

(b) [Icard is obliged to feed Oscar.]

According to Dancy, (a) totally grounds (b).[xxxi] But clearly (a) doesn't *necessitate* (b): certain other facts must be in place to *enable* (a) to ground (b). For example, it must be the case that that Icard's promise was not made under duress, that he is capable of feeding Oscar, and so on. These are *enabling* facts.

Back to twentieth century Dutch monarchs. Once the notion of enabling has been introduced, the following account is very natural:

The groundED fact:	$[(\forall x: M(x))\ F(x)]$
The facts that do the groundING:	[F(Wilhelmina)], [F(Juliana)], [F(Beatrix)]
The enabling fact:	$[(\forall x: M(x))(x=\text{Wilhelmina} \vee x=\text{Juliana} \vee x=\text{Beatrix})]$

More generally, we may say that if a, b, c, d, … are all the objects that have the property $\langle \lambda x : \alpha(x)\rangle$, then a fact of the form $[(\forall x: \alpha(x))\ \beta(x)]$ is totally grounded by the set of facts $\{[\beta(a)], [\beta(b)], [\beta(c)], \ldots\}$, and that the enabler for this grounding relation is $[(\forall x: \alpha(x))(x=a \vee x=b \vee x=c \vee \ldots)]$.[xxxii] For a very elegant discussion of this issue, see Chudnoff ms..

Equipped with the notion of enabling, we can replace the naïve version of grounding necessitarianism with a more refined version:[xxxiii]

> Suppose that some fact F is totally grounded by some set of facts Γ, with enablers Δ. Then there is no metaphysically possible world at which each element of $\Gamma \cup \Delta$ obtains and F doesn't obtain.

This refined version of grounding necessitarianism is consistent with Skiles' rules. At the same time, the idea that supervenience relations are to be explained by grounding relations is not endangered. The physicalist in the philosophy of mind may explain why the mental supervenes on the physical by saying that mental facts supervene on physical properties and that the enablers for these grounding relations are themselves physical facts. Similarly, the naturalist in metaethics may explain why the normative supervenes on the natural by saying that normative facts are grounded by natural facts, and that the enablers for these grounding relations are themselves natural facts. And so on. The resulting position has an additional benefit. Once we have rejected the crude form of grounding necessitarianism, we can revert to the 'naïve' rules for universal and existential quantification, adding that in the case of universal facts and negative existential facts the totality fact is an enabler.

We can then accept Rosen's natural identification of the totality fact with:

$[\forall x(x=a \vee x=b \vee x=c \vee \dots)]$ (where a, b, c… are all the things that there are)

without having to endorse to objectionable claim that the totality fact is an exception to the general principle about what grounds universal facts.

7. Improving the Schwartzkopff-Rosen Principle

We can now reformulate the Schwartzkopff-Rosen Principle to avoid the problems presented in section five. I write '$F \approx_R G$' for the conjunction of the following conditions:

$\text{Function}_{F,G}(R)$: $(\forall x{:}Fx)(\forall y_1{:}Gy)(\forall y_2{:}Gy)(Rxy_1 \wedge Rxy_2 \rightarrow y_1=y_2)$

$\text{Injective}_{F,G}(R)$: $(\forall y{:}Gy)(\forall x_1{:}Fx)(\forall x_2{:}Fx)(Rx_1y \wedge Rx_2y \rightarrow x_1=x_2)$

$\text{Total}_{F,G}(R)$: $(\forall x{:}Fx)(\exists y{:}Gy)\, Rxy$

$\text{Onto}_{F,G}(R)$: $(\forall y{:}Gy)(\exists x{:}Fx)\, Rxy$

I say that a binary relation is '$\text{Simple}_{F,G}$' if it takes the following form:[xxxiv]

$$\langle \lambda uv : (u=a_1 \wedge v=b_1) \vee (u=a_2 \wedge v=b_2) \vee (u=a_3 \wedge v=b_3) \vee (u=a_3 \wedge v=b_3) \vee \dots \rangle$$

where $a_1, a_2, \dots$ have the property F and $b_1, b_2, \dots$ have the property G. The new version of the Schwartzkopff-Rosen Principle is then as follows:

For all X, x, Y and y, if #X=x, #Y=y and x=y then each one of the facts [#X=#Y], [#X=x], [y=#Y] and [x=y] is totally grounded by $[(\exists W{:}\text{Simple}_{F,G}(W))\ F \approx_W G]$.

For all X, x, Y and y, if #X=x, #Y=y and x≠y then each one of the facts [#X≠#Y], [#X≠y], [x≠#Y] and [x≠y] is totally grounded by $[\neg(\exists W{:}\ \text{Simple}_{F,G}(W))\ F \approx_W G]$.

To illustrate this new version of The Schwartzkopff-Rosen Principle, I will look at what grounds [1=1]. Let F=⟨λx : x=0⟩ and let R be the relation ⟨λxy : x=0 ∧ y=0⟩. Then we have:

[1=1]		
←	[(∃W:Simple(W)) F $\approx_W$ F]	(S-R Principle)
←	[F $\approx_R$ F]	(2nd ∃ rule)
=	[Function(R) ∧ Injective(R) ∧ Total(R) ∧ Onto(R)]	(Def^n of '≈')
←	[Function(R)], [Injective(R)], [Total(R)], [Onto(R)]	(∧ rule)

Work through the grounds for these four facts, and you'll find that each is totally grounded by [0=0]. For example:

[Function(R)]		
=	[(∀x:Fx)(∀y_1:Fy)(∀y_2:Fy)(Rxy_1 ∧ Rxy_2 → y_1=y_2)]	(Def^n of 'Function')
←	[R00 ∧ R00 → 0=0]	(Skiles' rule, repeated)
←	[R00], [0=0]	(∧ rule, → rule)

Now:

[R00]		
=	[⟨λxy: x=0 ∧ y=0⟩00]	(Def^n of 'R')
←	[0=0 ∧ 0=0]	(λ rule)
←	[0=0]	(∧ rule)

So it remains only to explain what grounds [0=0]. Let G=⟨λx: x≠x⟩. Then:

[0=0]		
←	[(∃W:Simple(W)) G $\approx_W$ G]	(S-R Principle, since 0=#G)
←	[G $\approx_R$ G]	(2nd ∃ rule)
=	[Function(R) ∧ Injective(R) ∧ Total(R) ∧ Onto(R)]	(Def^n of '≈')
←	[Function(R)], [Injective(R)], [Total(R)], [Onto(R)]	(∧ rule)

It is a straightforward consequence of Skiles' rules that each of these facts is zero-grounded.

Indeed, it can be proven by mathematical induction that *all* facts of the form [x=x] and [¬x=y] where x, y ∈{0, 1, 2, … $\aleph_0$} are zero-grounded, so the regress problem is solved. Those who find the notion of zero-grounding mysterious may well reject the modified version of the Schwartzkopff-Rosen principle. Proponents of the principle may reply that it is a *desirable* feature of the principle that it implies that these facts are zero-grounded: this is an attractive way of making good on Agustín Rayo's claim that pure-mathematical claims are 'trivial'.[xxxv, xxxvi]

I started off by stating that the Schwartzkopff-Rosen Principle is attractive in part because it seems to imply that Aristotelian claim that arithmetical facts are grounded by empirical facts. We can now see that this statement requires one change. The proponent of the Schwartzkopff-Rosen Principle should say that every fact about cardinal number is either zero-grounded, or is grounded by some non-empty set of facts about the empirical world.

8. Autoarithmetic properties

Consider the property of *being one of Celia's favourite things*. There are five things which have this property, one of which is the number five. Now consider the property of *being a prime number less than six.* There are three things which have this property, one of which is the number three. Properties like this – properties which are instantiated by their own numbers – are 'autoarithmetic'. Both the original and the improved versions of the Schwartzkopff-Rosen Principle are problematic when applied to autoarithmetic properties. The simplest example is the property F=⟨λx : x=1⟩. Let R be the relation ⟨λxy : x=1 ∧ y=1⟩; then:

[1=1]	⇤	[(∃W:Simple(W)) F $\approx_W$ F]	
	⇤	[F $\approx_R$ F]	(2nd ∃ rule)
	=	[Function(R)∧Injective(R)∧Total(R)∧Onto(R)]	(Defn of '≋')
	⇤	[Total(R)]	(∧ rule)
	=	[(∀x:Fx)(∃y:Fy) Rxy]	(Defn of 'Total'.)
	⇤	[(∃y:Fy) R1y]	(Skiles' Rule)
	⇤	[R11]	(Skiles' Rule)
	=	[⟨λxy: x=1 ∧ y=1⟩11]	(Defn of 'R')
	⇤	[1=1 ∧ 1=1]	(λ Rule)
	⇤	[1=1]	(∧ rule)

Thus, the Schwartzkopff-Rosen Principle entails that there is a cycle of partial ground: the fact [1=1] is partially grounded by itself. This may seem problematic: it conflicts with a claim made in Fine 2010, Rosen 2010 and Schaffer 2009:

The acyclicality thesis: The relation of partial ground is acyclic.

A proponent of the Schwartzkopff-Rosen principle might respond by tinkering with the theory in order to make it consistent with the acyclicality thesis. For example, she might impose some system of types on the numbers, insisting that facts about a number of type *n* can only be grounded by facts about numbers of lower types. The reader should feel free to experiment with this – but I would like to propose a different approach. It is my view that the proponent of the Schwartzkopff-Rosen Principle should 'bite the bullet' and assert that there are cycles of partial ground. In support of this contention, she may appeal to the work of Elizabeth Barnes, who writes (ms.):

> [I]t's plausible that WWII just wouldn't have been the same event without the evacuation at Dunkirk. Without the evacuation at Dunkirk, it literally would've been a different war – the evacuation is an essential part of the war. But, similarly, we might think that being a part of WWII is essential to the evacuation of Dunkirk. Sure, you could have a duplicate of that event that doesn't take place in the wider context of WWII. But that duplicate isn't the evacuation at Dunkirk – part of what it is to be the evacuation at Dunkirk is to be a part of WWII. It's part of the character of the event that it had the goals it had, that it was part of a wider mission, that it took place within the particular geopolitical context that it did, etc. … The event of the evacuation depends on the event of WWII … But likewise the event of WWII depends on the event of the evacuation. An event that doesn't contain the evacuation at Dunkirk isn't WWII. The two events – WWII and Dunkirk – each depend on each other to be what they are.

So it is plausible that WWII 'ontologically depends' on the evacuation at Dunkirk and *vice versa.* I'm not asking you to accept this claim, but this is only one of several examples that Barnes discusses, and together these examples constitute a strong case against the assumption that ontological dependence is asymmetric. Now I suppose that if an object x is ontologically dependent on an object y, then [x exists] is partially grounded by [y exists]. Given this (surely very plausible) supposition, Barnes' examples show that the acyclicality thesis is less solid than it first appears.[xxxvii, xxxviii]

9. A summary of conclusions

We saw that the original version of the Schwartzkopff-Rosen Principle is not workable, because it does not cover a sufficiently wide range of arithmetical facts, and because it leads to an objectionable regress. My verdict on the modified version of the Schwartzkopff-Rosen Principle is less straightforward. I have uncovered no knock-down objection to the principle. At the same time, anybody who endorses the principle is burdened with a number of contentious theoretical commitments. In particular, a proponent of the modified version of the Schwartzkopff-Rosen Principle must reject necessitarianism while insisting that there are cycles of partial ground, and that some facts are zero-grounded. I suggest that anyone who finds the Schwartzkopff-Rosen Principle appealing, but who accepts orthodox views about grounding, must undertake a significant rethink.[xxxix]

I finish on a methodological note. Plausible general principles about grounding are ten a penny. However, detailed examination often reveals that these principles conflict with one another. So we must conclude that plausibility is at best an imperfect guide to truth. We should not be content to investigate general principles of ground one by one; we must look at groups of such general principles, investigating how they interact.

Appendix: A neo-Russellian theory of properties

In this appendix, I discuss an objection to these rules:

(Naïve 2nd ∀ rule)
If ∀X φ(X), then [∀X φ(X)] is totally grounded by the set of facts of the form [φ(X)].
If ¬∀X φ(X), then for any fact F of the form [¬φ(X)], [¬∀X φ(X)] ← F

(Naïve 2nd ∃ rule)
If ∃X φ(X), then for any fact F of the form [φ(X)], [∃X φ(X)] ← F.
If ¬∃X φ(X), then [¬∃X φ(X)] is totally grounded by the set of facts of the form [¬φ(X)].

(λ rule)
If ⟨λx: φ(x)⟩t, then [⟨λx : φ(x)⟩t]←[φ(t)].
If ¬⟨λx : φ(x)⟩t, then [¬⟨λx : φ(x)⟩t]←[¬φ(t)].

The problems that I discuss here arise in the same way for the 'sophisticated' versions of the 2nd ∀ rule and the 2nd ∃ rule. However, for simplicity, and in light of the discussion in section six, I will restrict my discussion to the 'naïve' versions of these rules. Consider:

[⟨λx : ∃X Xx⟩a]
⇤ [∃X Xa] (λ rule)
⇤ [⟨λx : ∃X Xx⟩a] (2nd ∃ rule)

This establishes that the proposed account of the metaphysics of second-order quantification entails that there is a cycle of partial ground. One might seek to avoid the conclusion that there are cycles of partial ground by modifying the 2nd ∃ rule in this way:[xl]

If ∃X φ(X), then for **some** fact F of the form [φ(X)], [∃X φ(X)] ← F.

However, there is an analogous problem involving universal second-order quantification that is not so easily solved.

Consider the fact [⟨λx : ∀X(Xx ∨ ¬Xx)⟩a]. The λ rule tells us that this fact is grounded by [∀X(Xa ∨ ¬Xa)]. The second ∀ rule tells us that this fact is grounded by the set of all facts of the form [Xa ∨ ¬Xa], and this includes:

[⟨λx : ∀X (Xx ∨ ¬Xx)⟩a ∨ ¬⟨λx : ∀X (Xx ∨ ¬Xx)⟩a]

The ∨ rule tells us that this fact is grounded by [⟨λx : ∀X (Xx ∨ ¬Xx)⟩a]. We have derived a cycle in the ground for [⟨λx : ∀X (Xx ∨ ¬Xx)⟩a], and there's no natural way of weakening the 2nd ∀ rule to avoid this cycle.[xli] Perhaps some philosophers will simply accept that the grounding relation is cyclic in this way. But before endorsing this conclusion we should explore the alternatives.

I call this the 'puzzle of property grounding'; it is analogous to the 'puzzles of ground' discussed in Fine 2010 and Krämer 2013. Fine suggests several solutions to his puzzles: the reader should feel free to investigate whether Fine's solutions to his puzzles can be adapted elegantly to solve the puzzle of second-order quantification. I wish briefly to describe a different approach,[xlii] which I call 'neo-Russellian'.[xliii] Here's the basic idea. Properties, on the neo-Russellian view, are distinguished by 'level'. There are zeroth level properties, first level properties, second level properties … and more generally there are n^{th} level properties for each natural *n*.[xliv] Each predicate and second-order variable must be assigned a level, which will be represented using a superscript numeral. For example, 'X^{13}' ranges of thirteenth level properties. I'll use superscript numbers to represent the level of the relevant property when using the λ-notation too. For example, an expression of the form ⌜ $\langle\lambda^{23} x : \varphi(x)\rangle$ ⌝ refers (if at all) to a twenty-third level property.

In short, the idea is that facts of the form [X^n y] (where $n>0$) are grounded in facts involving properties at levels below *n* – thus cycles of partial ground are avoided. Let's take a look at the components of the neo-Russellian theory in more detail.

First Component: Quantificational Involvement

There is an obvious distinction to be drawn between open formulas which contain quantifiers and open formulas which don't contain quantifiers. The formulas which do contain quantifiers can be further classified according to *which* quantifiers that they contain. The first component of the neo-Russellian theory is that there are parallel distinctions to be drawn between different sorts of properties. For example, the property of *being an owl smaller than some owl* involves first-order existential quantification. The property of *having all the first level properties that Barney has* involves universal quantification over first level properties.

Second Component: A Restriction on Property Existence

As I said, the neo-Russellian view is that that facts of the form [X^n y] (where n>0) are grounded in facts involving properties at levels below n. In order to secure this, we impose the following condition on property existence:

> Zeroth level properties do not involve second-order quantification. For $n \geq 1$, n^{th} level properties involve quantification at level (n – 1), and perhaps at lower levels, but not at level n or above.

For example, the following λ-expression does not represent a property, according to the neo-Russellian theory:

$\langle \lambda^0 x: \exists X^1\, X^1 x \rangle$

Third Component: Adapting the Rules

It is easy to modify (the λ rule), (the 2nd ∀ rule) and (the 2nd ∃ rule) in accordance with the idea that properties are distinguished by level:

(The new λ rule)
If $\langle \lambda^n x : \varphi(x)\rangle t$, then $[\langle \lambda^n x : \varphi(x)\rangle t] \leftarrow [\varphi(t)]$.
If $\neg\langle \lambda^n x : \varphi(x)\rangle t$, then $[\neg\langle \lambda^n x : \varphi(x)\rangle t] \leftarrow [\neg\varphi(t)]$.

(The new naïve 2nd ∃ rule)
If $\exists X^n\, \varphi(X)$, then for each fact F of the form $[\varphi(X^n)]$, $[\exists X^n\, \varphi(X^n)] \leftarrow$ F.
If $\neg\exists X^n\, \varphi(X^n)$, then $[\neg\exists X^n\, \varphi(X^n)]$ is grounded by the set of all facts of the form $[\neg\varphi(X^n)]$.

(The new naïve 2nd ∀ rule)
If $\forall X^n\, \varphi(X^n)$, then $[\forall X^n\, \varphi(X^n)]$ is totally grounded by the set of all facts of the form $[\varphi(X^n)]$.
If $\neg\forall X^n\, \varphi(X^n)$, then for each fact F of the form $[\neg\varphi(X^n)]$, $[\neg\forall X^n\, \varphi(X^n)] \leftarrow$ F.

The reader can easily check that these new rules block the derivation of cycles of partial ground, thus solving the problem.

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[i] The quote is from Rosen 2010 – though it should be noted that Rosen's paper was widely read and discussed long before it was eventually published.

[ii] Wilson 2014 is a particularly perceptive critical discussion.

[iii] Other early proponents of the notion of grounding include Dancy (e.g. Dancy 1981), DePaul (e.g. DePaul 1987), Fine (e.g. Fine 1010), and Schaffer (e.g. Schaffer 2009). See Berker (ms.) for a perceptive history of the discussion.

[iv] Rosen (2010) says that the arguments of the grounding relation are *facts;* Schaffer (2009) allows that objects, properties and other things may also stand in the grounding relation.

[v] Fine (2012) uses a sentential connective (' … *because* …') rather than a predicate to express claims about grounding.

[vi] See, for example, Jenkins 2011 on the question of whether the grounding relation is asymmetric. Barnes (ms.) discusses whether the relation of ontological dependence is asymmetric. Cameron 2008 is an analysis of whether ontological dependence is well-founded. Schaffer (2012) discusses whether the grounding relation is transitive.

[vii] See Fine 2012 and Bennett 2011 for some discussion of this question.

[viii] Rosen 2010, pg. 123; Schwartzkopff 2011, pg. 362.

[ix] This is the theme of *Metaphysics* M, ch. 2-3.

[x] The differences between Fine's framework and Rosen's are fairly small and I will draw heavily on both Fine and Rosen. Schaffer's views are rather different. Since Schaffer's formal framework has only recently been published, it has so far been less discussed.

[xi] Some of the principles discussed in the paper are schematic. In these schemata, I use lower case Greek letters (α, β, φ, …) to stand in for sentences. A Greek letter with parenthetic variables after it stands in for formulas in which no variable other than those parenthesized occurs free. For example, '$\alpha(x)$' stands in for formulas with no free variables except perhaps 'x'. I will use the expressions 't', 't_0', 't_1', 't_2', … to stand in for names. I write '$\varphi(t_1, \ldots, t_n)$' to stand in for the result of substituting the name corresponding to t_i for all free occurrences of x_i in the formula $\varphi(x_1, \ldots, x_n)$.

[xii] The assumption that the groundING facts form a set is not to be taken too seriously. I suppose that the fact [$\forall$x x=x] is grounded by *all* facts of the form [x=x], and I suppose that there are too many

such facts to form a set. The assumption that the grounding facts form a set is a useful but dispensable pretence.

[xiii] See Dasgupta 2014 for a defence of the non-standard view that the grounding relation can take several arguments on the left.

[xiv] As Rosen (2010) comments, if 'the most fundamental relation in the vicinity' is not transitive, we may shift our attention to its transitive closure. Schaffer (2012) rejects Rosen's version of the transitivity thesis. He then develops a 'contrastivist' theory of ground, which includes a more elaborate version of the transitivity thesis. My account of what grounds numerical facts is not contrastivist, but a contrastivist version of the account could be developed.

[xv] Fine includes the totality fact in the total ground for *all* first-order quantified facts, not just universal and negative existential facts. Rosen offers the Sophisticated 1st ∀ rule as an account of what grounds 'accidental' universal facts only; he has a different account of what grounds non-accidental universal facts. See endnote 39 for more discussion of this point.

[xvi] Later I will cast doubt on the claim that there can be no cycles of partial ground. See section eight.

[xvii] He adds (pg. 121, fn. 13): 'If the existence of one thing can be grounded in the existence of others, then the totality fact need not be basic. The basic fact in the vicinity might simply itemize the *ontologically fundamental* items and then assert the completeness of the inventory.'

[xviii] Fine 2012 contains a more elaborate discussion of λ-abstraction, which involves a distinction between two different sorts of λ-abstraction. The arguments in the rest of the paper could all easily be adapted to accommodate Fine's theory.

[xix] It has been suggested to me that $[\neg\langle\lambda x : \varphi(x)\rangle t]$ grounds $[\neg\varphi(t)]$ and not *vice versa.* I reject this proposal, because it leads immediately a version of Bradley's regress (Bradley 1969).

[xx] The neo-fregean movement was initiated by Wright (1983). For more recent work see Hale and Wright 2001.

[xxi] Rosen and Schwartzkopff only state the first of these two claims – they offers an account of what grounds facts of the form [#X=#Y] without also offering an account of what grounds facts of the form [#X≠#Y]. I take it that the Schwartzkopff-Rosen Principle is a natural extension of Rosen and Schwartzkopff's weaker claim.

xxii Schwartzkopff uses plural quantification in place of second-order quantification. His principle is roughly:

> For all xx and yy, if the number of xx is identical to the number of yy, then this fact is totally grounded by the fact that there is a relation which pairs xx and yy one-to-one.

I prefer to follow Rosen in using second-order quantification, for three reasons. First, the 'second-order version' of Hume's Principle is more familiar than the 'plural version'. Second, if we assume (*pace* Schein 2006) that there is no empty plurality, Schwartzkopff's principle gives us no account of what grounds fact involving zero. Third, I take myself to have an adequate account of what facts involving quantification over properties (see the appendix) but as yet there is no theory of what grounds facts that involve plural quantification.

Schwartzkopff defends his use of plurals by considering sentences involving 'collective' predicates, such as, 'The number of Greeks who surround Troy is 1000.' Schwartzkopff is quite right that this sentence should not be rendered:

> #⟨λx: x is a Greek who surrounded Troy⟩ = 1000

No individual Greek had the property of being a Troy-surrounder. However, we can deal with the original sentence as follows:

> #⟨λx: x is one of the Greeks who together surround Troy⟩ = 1000.

So Schwartzkopff's example does not force us to abandon the traditional second-order approach.

xxiii Should neofregeans accept the Schwartzkopff-Rosen Principle? I suspect not. A recurring theme in neofregean writing is that, for example, '#*FlamingoSpecies* = #*WelshCity*' and '∃W *FlamingoSpecies* $\sim_W$ *WelshCity*' have the same content. If this is right, then presumably [#*FlamingoSpecies* = #*WelshCity*] and [∃W *FlamingoSpecies* $\sim_W$ *WelshCity*] are the very same fact – not distinct facts one of which grounds the other. This proposal is subject to many of the same difficulties as the Schwartzkopff-Rosen Principle.

For discussion of the sense in which '#*FlamingoSpecies* = #*WelshCity*' and '∃W *FlamingoSpecies* $\sim_W$ *WelshCity*' have the same content, see Hale (2001).

xxiv One shortcoming of the proposal is that it leaves untouched the question of what grounds facts like:

> [Julius Caesar ≠ 2]

I will not consider this question here: it is discussed in some detail in Rosen and Yablo (ms.)

[xxv] A reviewer at Noûs has kindly pointed out a pair of alternative approaches, which I will now briefly discuss.

The reviewer's first suggestion is that each cardinal number κ is associated with a canonical property F_κ, such that:

$$[\kappa = \kappa] \leftarrow [\exists W\, F_\kappa \sim_W F_\kappa]$$
$$[\kappa \neq \mu] \leftarrow [\neg\exists W\, F_\kappa \sim_W F_\mu]$$

The problem with this approach, of course, is that it is hard to see what these "canonical properties" could be. For the natural numbers, the following proposal may be plausible:

$$F_0 = \langle \lambda x.\ x \neq x \rangle$$
$$F_1 = \langle \lambda x.\ x = 0 \rangle$$
$$F_2 = \langle \lambda x.\ x = 0 \vee x = 1 \rangle$$
$$F_3 = \langle \lambda x.\ x = 0 \vee x = 1 \vee x = 2 \rangle$$

…

…

However, this proposal has no natural generalization to the infinite cardinals.

The reviewer's second proposal is this. For each cardinal κ, let "$\exists!_\kappa x\, \varphi(x)$" mean *there are exactly κ objects x such that $\varphi(x)$*. Then, plausibly:

$$[\kappa = \kappa] \leftarrow [\forall X \forall Y\, ((\exists!_\kappa u\, Xu \wedge \exists!_\kappa u\, Yu) \rightarrow \exists W(X \sim_W Y))]$$
$$[\kappa \neq \mu] \leftarrow [\forall X \forall Y\, ((\exists!_\kappa u\, Xu \wedge \exists!_\mu u\, Yu) \rightarrow \neg\exists W(X \sim_W Y))]$$
$$[\kappa = \#F] \leftarrow [\forall X\, (\exists!_\kappa u\, Xu \rightarrow \exists W(X \sim_W F))]$$
$$[\kappa \neq \#F] \leftarrow [\forall X\, (\exists!_\kappa u\, Xu \rightarrow \neg\exists W(X \sim_W F))]$$

This approach is very elegant, and I am happy to endorse it as an alternative to the proposal made in the main text. (Of course, a thorough development of the proposal would have to include an account of what grounds facts of the form $[\exists!_\kappa u\, Xu]$.)

[xxvi] See for example Schaffer 2010.

[xxvii] See Sider 2011, 8.2.2 for a slightly less exotic example which motivates the idea that the grounding relation might not be well-founded. See Cameron 2008 for another take on the issue.

[xxviii] In particular, one might accept Skiles' rule for non-vacuous cases only, giving some other account of what grounds vacuous universal restricted generalizations. Mike Raven and Adam Lovett have each

informed me that Skiles did not originally intend his account to apply in the case of vacuous universal restricted generalizations.

[xxix] Mike Raven has made the following objection to my proposal. Consider the following two facts:

(i) [(∀x: *TwentiethCenturyFrenchMonarch*(x)) *BanjoPlayer*(x)]

(ii) [(∀x: *ChildofSimoneDeBeauvoir*(x)) *Tall*(x)]

It is a consequence of Skiles' account that (i) and (ii) have the same total ground (viz. the empty set). Raven protests, however, that intuitively (i) and (ii) have *different* explanations. Specifically, it seems that the proper explanation of (i) is that there are no twentieth century French monarchs while the proper explanation of (ii) is that Simone de Beauvoir had no children. So there must be something wrong with Skiles' account.

In response, I suggest that while (i) and (ii) have the same ground, the enablers in the two cases are different – and this is quite adequate to capture the intuition that (i) and (ii) have different explanations.

[xxx] Trogdon (2013) presents a very different argument for grounding necessitarianism. The argument deserves study, but I lack space to discuss it properly here.

[xxxi] What I am calling the 'grounding' relation, Dancy calls 'resultance'.

[xxxii] If nothing satisfies α(x), I suggest, then [(∀x: α(x)) β(x)] is zero-grounded and the enabler is [(∀x: α(x))⊥].

[xxxiii] Admittedly, my 'refined' version of grounding necessitarianism may require further refinements. In particular, I have not incorporated into my account Dancy's idea that there are *disablers* as well as enablers. I should add that I do not mean to suggest that Dancy would approve of my use of his work on enabling.

[xxxiv] This is rather misleading, because it suggests that the field of a simple relation must be countable – this is not what I intend. It would perhaps be better to define the term 'simple' as follows: a $\text{simple}_{F,G}$ relation is a disjunction of arbitrarily many relations of the form ⟨λuv : u=a ∧ v=b⟩, where a has the property F and b has the property G.

[xxxv] For discussion of trivialism, see Rayo 2009 and Rayo 2013. Rayo himself does not use the notion of grounding in this way.

[xxxvi] I suspect that to give an adequate treatment of cardinals greater than $\aleph_0$, we will have to appeal to properties of other mathematical objects. For example, we may say that [$\aleph_1=\aleph_1$] is grounded by the fact that the identity relation is a one-to-one pairing of the countable ordinals with themselves.

[xxxvii] I should stress that Barnes herself does *not* claim that there are cycles of partial ground; she focuses on ontological dependence.

[xxxviii] Schaffer insists that the relation of ground is acyclic (Schaffer forthcoming, sect. 1.3). However, it's worth noting that nothing in Schaffer's formalism forces this conclusion. Indeed, Schaffer bases his formalism on Halpern 2000, and Halpern explicitly permits cycles.

[xxxix] I have been assuming throughout that a universal generalization is partially grounded by each of its instances. However, in Rosen 2010 it is proposed that certain 'non-accidental' generalizations are grounded in facts about essences. Rosen offers the following as a plausible example:

[All triangles have three sides.] $\leftarrow$ [It lies in the nature of triangularity that all triangles have three sides.]

A reviewer at Noûs has kindly proposed that of the regresses described in the text can be blocked by insisting that relevant generalizations are grounded, not be their instances, but by facts about the essences of logical objects. The proposal is intriguing. However, further work is needed on the essences of logical objects before it can be properly evaluated.

[xl] A manoeuvre like this is suggested in Fine 2010.

[xli] Worse, we have no 'completely satisfactory' explanation for [$\langle\lambda x : \forall X(Xx \vee \neg Xx)\rangle a$], in the sense of Fine (2010).

[xlii] I think that when Fine (2010) mentions a 'predicativist' approach to his puzzles, he has something like my neo-Russellian proposal in mind. If so, then the point of the rest of this appendix is to spell out what is left unsaid in Fine 2010.

[xliii] The theory is 'neo-Russellian' because it is reminiscent of theories developed by Russell in response to the set-theoretic and semantic paradoxes. See, e.g., Russell 1908.

[xliv] One might insist that there are also properties of type ω, properties of type ω+1, properties of type ω+2 … and so on through all the ordinal numbers. But I won't pursue this idea here.