'The Construction of Logical Space', by Rayo, Agustín

Oxford: Oxford University Press, 2013, pp. xix + 220, £35 (hardback).

Not so long ago, 'positivist' was one of those words (like 'Cartesian' and 'reductionist') that academics sometimes use when professional etiquette prevents them from using a more quotidian slur term. But now Carnap especially is back in fashion. Agustín Rayo has described his new book as a 'revival of logical empiricism', and his title is an allusion to the *Aufbau*. Rayo's central topics are the epistemology of modality (chapters 2, 5 and 6) and of mathematics (chapters 3, 4, 7 and 8). The first chapter is devoted to a discussion of the 'just is' operator—which Rayo himself introduced in earlier work [Rayo 2009]. Here is one of his examples, slightly adapted [3]:

> To be composed of water just is to be composed of H_2O.

This can be paraphrased, 'there is no difference between being composed of water and being composed of H_2O', or 'the property of being composed of water is identical to the property of being composed of H_2O' (though Rayo has some reservations about reference to properties in this context, see [68]). The operator can also be used to identify relations with various arities:

> For x to be taller than y just is for y to be shorter than x.

> For x to be the child of y and z just is for y and z to be the parents of x.

The following exemplifies a rather different use of the expression 'just is':

> For it to be the case that Sydney is in New South Wales just is for it to be the case that New South Wales contains Sydney.

Rayo would paraphrase this as follows:

> 'Sydney is in New South Wales' has the same truth condition as 'New South Wales contains Sydney'.

Rayo argues that when deciding which 'just is'-statements to accept, we should use a sort of cost–benefit analysis. The cost of accepting that to be an F just is to be a G is that one loses a potentially useful distinction—viz. the distinction between being an F and being a G. The benefit is that one excuses oneself from having to answer certain questions, which might otherwise turn out to be problematic. In particular, there is no need to answer the question 'Why is every F a G and *vice versa*?' if to be an F just is to be a G.

Here is an example. Consider a theist, who is convinced that God approves of all right actions, and of no actions which are not right; the theist considers:

> For an action to be right just is for it to be approved of by God.

If she accepts this 'just is'-statement, she won't have to explain why God approves of all and only those actions which are right. If such an explanation would be hard to provide, this is an important benefit. On the other hand, if she accepts the 'just is'-statement, she'll lose the distinction between *being right* and *being approved of by God*. This could be an important cost. For example, suppose that she's asked to explain why God deserves our praise. It will be tempting for her to reply that God deserves our praise, in part, because He only ever does what's right. But of course, if to be right just is to be approved of by God, this just amounts to the claim that God approves of all of his own actions—and smugness is not a praiseworthy quality. I'd say it's not clear, in the absence of further work, whether the theist should accept this 'just is'-statement.

We can distinguish 'realist' from 'pragmatist' versions of Rayo's account (these are not Rayo's terms). According to a realist, some 'just is'-statements are objectively true, and some objectively false. Pragmatists deny that 'just is'-statements have objective truth values; instead, they claim, one's choice of 'just is'-statements is a 'practical matter' which 'cannot be fully separated from the aims of a particular community' [63]. According to the pragmatist, invention is the mother of necessity. In earlier work, Rayo committed himself to the pragmatist position [Rayo 2009], but in the current book he is more cautious. He clearly *prefers* the pragmatist position, saying that he doesn't know how to make sense of the distinction between objectively true and objectively false 'just is'-statements [57–8]. However, he leaves open the possibility that someone in the future will find a way of making sense of this distinction, thereby vindicating realism.

Having used Rayovian cost-benefit analyses to figure out which 'just is'-statements to accept, one can derive conclusions about metaphysical necessity from them. Rayo's discussion of the connections between 'just is'-statements and statements about necessity is elaborate, but the reader can get some idea of his approach by considering the following schema, which Rayo endorses:

$$\Box \forall x(Fx \leftrightarrow Gx) \text{ iff to be an F just is to be a G.}$$

I suspect that the right-to-left direction of this equivalence will prove uncontroversial. The left-to-right direction is more problematic. Consider, for example, non-naturalist views in metaethics, such as that of Parfit [2011]. Most metaethical non-naturalists are willing to agree, for example, that 'morally permissible' is necessarily coextensive with some (perhaps highly complex) naturalistic predicate, *F* say; but it seems that to consent to ⌜ to be morally permissible just is to be *F* ⌝ would be to renounce non-naturalism. So some readers will find Rayo's epistemology of necessity unsatisfying.

Rayo's Carnapian sympathies are nowhere more evident than in his discussion of mathematics. The book contains an extended defence of 'trivialism': the claim that truths of pure mathematics have 'trivial' truth-conditions, in the sense that 'nothing is required of the world in order for the truth-conditions of a mathematical truth to be satisfied' [98]. More precisely, the claim is that if *S* is a true pure-mathematical sentence, the truth condition of *S* is the same as the truth condition of '$\forall x \, x=x$'; for Rayo, this is equivalent to:

⌜ For it to be the case that *S* just is for it to be the case that $\forall x \, x=x$. ⌝

Trivialism is reminiscent of Carnap's claim that mathematical truths (being analytic) have 'no real content' [Carnap 1934: xiv], and are not 'genuine statements about reality' [ibid.: 41].

Rayo's epistemology of mathematics is also similar to that of some of the logical empiricists. He argues that pure-mathematical knowledge can be obtained using the following procedure:

> *Step One*: Write down some 'axioms', which contain only logical particles and terms hitherto unused.

> *Step Two*: Come to know that the axioms are consistent. (If they are not consistent, start again at *Step One*.)

> *Step Three*: Stipulate that the non-logical terms in the axioms are to be interpreted in such a way that the axioms are trivial truths.

> *Step Four*: Introduce some further terms by explicit definition.

> *Step Five*: Prove some theorems by deduction from the definitions.

For example, one might introduce the terms 'natural number', 'zero', 'plus', 'times' and 'successor' by stipulating that the Peano axioms are trivial truths. One could then introduce further terms ('prime', 'even', and so on) using explicit definitions. Having confirmed that the axioms are consistent, one will be in a position to learn the theorems of Peano arithmetic by deducing them from the axioms.

Rayo doesn't say explicitly that using this method one can achieve *a priori* knowledge; however, he also does not suggest that empirical information will be needed when carrying out this procedure, and so I think we can tentatively attribute to Rayo the claim that pure mathematical truths are *a priori* in the sense that they are knowable independently of experience. A more or less standard objection to this sort of view goes as follows:

> I'm willing to concede that (for example) the axioms of Peano arithmetic are 'definitions'. But it doesn't follow that the Peano axioms are actually true. At best, what follows is that the Peano axioms are true *if numbers exist*. Here's an analogy. It might be that the definition of 'God' is 'God is a perfect being who created the world'. But it doesn't follow that this sentence is true. At most, it follows that *if God exists*, He is a perfect being who created the world. (Based on Field 1984.)

In response to this sort of objection, Rayo develops an argument (too long, unfortunately, to rehearse here) for the claim that when one stipulates that one's axioms are to be trivially true, one's stipulation *will* succeed provided that (i) the axioms are consistent, and (ii) they contain only logical particles and the newly defined terms. I find Rayo's discussion of this point convincing, but some challenges remain. One challenge is that of explaining how *Step Two* could be carried out. A second challenge is this. Whatever one makes of Rayo's claim that mathematical knowledge *could in principle* be achieved using the method described, it is clear that this is not how in actual fact mathematical research generally advances. When one uses Rayo's method, one defines one's

terms *before* any theorems are established; however, it is not uncommon in the history of mathematics for important theorems to be established before their component terms have been adequately defined. So the question of how real mathematicians acquire mathematical knowledge has not yet fully been answered.

I have not been able to summarize all parts of the book in this short review. Rayo also includes a chapter on the 'deduction problem' for Stalnakerian theories of content (see Stalnaker [1984: ch. 5]), and a chapter on the question of to what extent it's possible to 'paraphrase' mathematical statements in non-mathematical language. What's amazing is that Rayo manages to fit all of this (and more) into only about 200 small pages of unhurried text. The resulting theory is attractive in many ways, and remarkable in its scope and unity. Anyone who works on modality, mathematics or meta-metaphysics should add this book to her 'read ASAP' list; more broadly, anyone in 'core' philosophy who reads the book will benefit from it.

References

1. Carnap, Rudolf 1934 (1937). The Logical Syntax of Language, trans. Amethe Smeaton, London: Kegan Paul.

2. Carnap, Rudolf 1928. Der logische Aufbau der Welt, Berlin: Bernary.

3. Field, Hartry 1984. Platonism for Cheap? Crispin Wright on Frege's Context Principle. Canadian Journal of Philosophy 14/4: 637–62.

4. Parfit, Derek 2011. On What Matters. Oxford: Oxford University Press.

5. Rayo, Agustín 2009. Towards a Trivialist Account of Mathematics, in New Waves in Philosophy of Mathematics, ed. Otávio Bueno and Øystein Linnebo, Hampshire and New York: Palgrave Macmillan.

6. Stalnaker, Robert 1984. Inquiry. Cambridge: Cambridge University Press.